Assignment 11

Coverage: 16.7

Exercises: 16.7 no. 3, 6, 4, 8, 13, 16, 18.

No homework.

Supplementary Problems

1. Let S be the triangle with vertices at $(1,0,0),(0,2,0),(0,0,7)$ with normal pointing upward. Find the circulation of the vector field $\mathbf{F}=x\mathbf{i}+3z\mathbf{j}$ around the boundary of S with the orientation determined by the chosen normal of S.

2. Show that for a closed oriented surface S, that is, a surface without boundary,
$$\iint_S \nabla\times\mathbf{F}\cdot\mathbf{n}\,d\sigma=0\ .$$
Hint: See how to apply Stokes' theorem.

3. (Optional) Let S be the surface given by $(x,y)\mapsto(x,y,f(x,y)),(x,y)\in D$. That is, it is the graph of f over the region D. Show that in this case Stokes' theorem
$$\iint_S \nabla\times\mathbf{F}\,d\sigma=\oint_C \mathbf{F}\cdot\,d\mathbf{r}\ ,$$
($\mathbf{F}$ is a smooth vector field on S) can be deduced from Green's theorem for some vector field on D. Hint: Let the boundary of D be $\mathbf{r}(t)=(x(t),y(t))$. Then the boundary of S is $\mathbf{c}(t)=(x(t),y(t),f(x(t),y(t)))$. Convert the integration in S and C to the integration on D and the boundary of D respectively.